FAST MEASUREMENT OF CAVITY COUPLING COEFFICIENTS

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Abstract. A swept frequency technique is described which allows to quickly obtain accurate values for the coupling coefficients of transmission-type cavities. An illustrative example is presented, and conditions for the validity of the new method are established.

Recently, a novel technique for cavity parameter measurement has been developed. This technique is based upon the evaluation of xy-plots generated by simultaneously frequency sweeping of both the cavity under test and a tunable reference cavity /1/-/3/. Based upon a theory scrutinized in /1/, and applicable to all types of cavities, general procedures for high-precision determination of resonant frequency, of loaded Q-factor, and of coupling coefficients were reported in /2/. To shorten measuring times, a quick measurement of resonant frequency derived from the general technique was described in /2/, whereas /3/ dealt with a fast Q-measurement procedure applicable to transmission-type resonators. This note describes an accurate, but quickly to perform measurement of the coupling coefficients of transmission-type resonators.

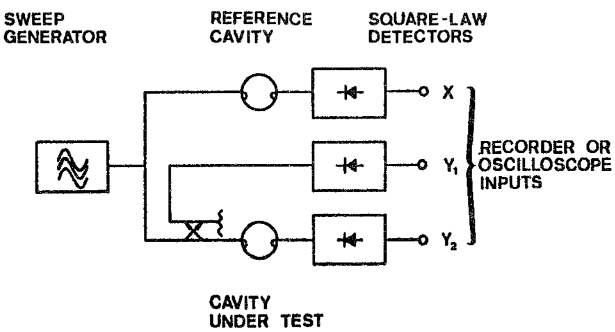

Fig. 1: Basic experimental set-up

Figure 1 shows the basic experimental set up: Both microwave resonators, the reference cavity and the cavity under test, are simultaneously frequency swept by a leveled sweep oscillator. A signal proportional to the microwave power transmitted through the reference cavity is fed to the x-input of a plotter or of an oscilloscope, whereas signals proportional to the power reflected from and transmitted through the cavity under test are fed to the y_1- and y_2-input, respectively. When the resonant frequencies of both resonators nearly coincide, closed loop curves are plotted which can be described analytically /2/ by

$$y_1 = k_2\left[1 - \frac{b}{1+q^2(c\pm\sqrt{k_1/x -1})^2}\right] , \qquad (1.a)$$

$$y_2 = \bar{k}_2\left[\frac{\bar{b}}{1+q^2(c\pm\sqrt{k_1/x -1})^2}\right] . \qquad (1.b)$$

In (1) the following abbreviations have been used

$$b = 4\beta_1(1+\beta_2)/(1+\beta_1+\beta_2)^2 , \qquad (2.a)$$

$$\bar{b} = 4\beta_1\beta_2/(1+\beta_1+\beta_2)^2 , \qquad (2.b)$$

where β_1 and β_2 are the input- and output-coupling coefficients of the cavity under test, respectively. The quantity c is a normalized tuning parameter given by

$$c = 2Q_{L1}(f_1-f_2)/f_2 , \qquad (3)$$

and the normalized Q-factor q is defined by

$$q = Q_{L2}/Q_{L1} , \qquad (4)$$

where Q_{L1} (Q_{L2}) and f_1 (f_2) denote loaded Q-factor and resonant frequency of the reference (unknown) cavity. Finally, k_1, k_2 and $\bar{k}_2$ are scaling factors accounting for the deflection sensitivities of the x-, y_1-, and y_2-channels, respectively.

To achieve the same scale for both y_1- and y_2-plot, i.e. to accomplish $k_2 = \bar{k}_2$, the plotter or oscilloscope must be calibrated in the following way: The unknown resonator is replaced by a short circuit, thus producing the maximum possible y-deflection, y_{1max}, in the y_1-plot. Removing the short circuit and replacing it by the y_2-detector causes a deflection y_{2max} proportional to the maximum transmitted power. Adjusting now the sensitivity of the y_2-channel (and thus adjusting $\bar{k}_2$) such that $y_{2max}=y_{1max}=y_{max}$ holds, yields the desired calibration $k_2 = \bar{k}_2$. In order to obtain a correct calibration for the whole sweep range, the y_1- and y_2-detectors should be a matched pair.

After inserting the unknown cavity according to Fig. 1, we now can plot a pair of y_1- and y_2-curves as shown in Fig. 2b. These two graphs intersect each other at four points determined by the four possible solutions x_s of

$$1- \frac{b}{1+q^2(c\pm\sqrt{k_1/x_s -1})^2} = \frac{\bar{b}}{1+q^2(c\pm\sqrt{k_1/x_s -1})^2} . \qquad (5)$$

For the intersection points in mind, we solve (5) for the case of equal signs in the denominators of (5), leading to

$$1+q^2(c\pm\sqrt{k_1/x_s -1})^2 = b + \bar{b} . \qquad (6)$$

Introducing (6) into (1) yields the same normalized y-coordinate, y_s/y_{max}, for both intersection points,

$$y_s/y_{max} = \bar{b}/(b+\bar{b}) . \qquad (7)$$

After insertion of (2), equation (7) can easily be solved for the unknown output-coupling coefficient β_2

Manuscript received August 18, 1977; revised January 3, 1978.

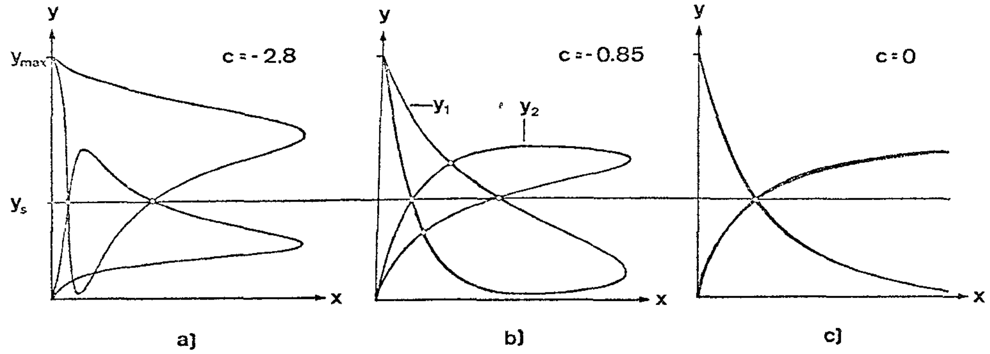

Fig. 2: Plots taken with a set-up according to Fig. 1 for several values of the normalized tuning parameter c. The output-coupling coefficient β_2 of the cavity under test is obtained from the c-independent ordinate y_s of two intersection points of the y_1- and y_2-plots alone.

$$\beta_2 = \frac{y_s/y_{max}}{1-2y_s/y_{max}} \quad . \tag{8}$$

Equations (7) and (8) reveal the extreme simplicity of the coupling-coefficient measurement procedure: It is only necessary

(a) to plot a pair of y_1- and y_2-curves, and

(b) to determine the normalized y-coordinate, y_s/y_{max}, of the two intersection points located at equal ordinates.

Note that it is not necessary to know any other cavity parameter, not even of the reference cavity, which greatly contributes to the simplicity of the method.

As can be seen from (7), y_s/y_{max} is independent of the tuning factor c. Therefore, the desired intersection points will be readily found traveling on a straight line parallel to the x-axis when tuning the resonant frequency f_1 of the reference cavity. This is illustrated by Fig. 2 which shows several plots taken with an X-band set up for various values of the tuning parameter c. From these plots, $y_s/y_{max} = 0.40$ and hence $\beta_2 = 2.11$ is determined. The unknown input-coupling coefficient β_1 may be obtained by simply reversing the unknown cavity (= by interchanging the input and output ports) and repeating the measurement procedure outlined above.

Finally we will establish the conditions for the occurence of intersection points. It is evident that the minimum of the y_1-plot must be equal or less than the maximum of the y_2-plot. Therefore, we can write and "intersection-point condition"

$$\bar{b} \geqslant 1 - b \ , \tag{9}$$

or, in terms of the coupling coefficients,

$$4\beta_1\beta_2 \geqslant (1-\beta_1+\beta_2)^2 \quad . \tag{10}$$

Equation (10) describes the area inside a hyperbola in the β_1-β_2-plane and is depicted in Fig. 3. The area hatched parallel to the β_1-axis includes all combinations of β_1 and β_2 for which intersection points occur. Reversing the cavity, i.e. interchanging indices "1" and "2" in (10), leads to an analogous area of allowed coupling coefficients, hatched parallel to the β_2-axis (Fig. 3). Consequently, the cross hatched area includes all values of β_1 and β_2 for which intersection of the y_1- and y_2-curves occur independently of the orientation of input and output ports of the unknown cavity.

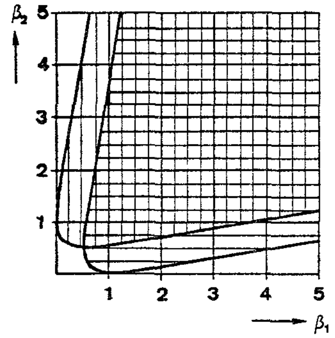

Fig.3: Allowed areas of input- and output-coupling coefficients β_1 and β_2. The area hatched parallel to the β_1-axis indicates all combinations of β_1 and β_2 for which intersection points occur; the same is true for the area hatched parallel to the β_2-axis when the cavity under test is reversed.

The presented method for coupling coefficient measurement completes the quick resonant frequency measurement procedure described in /1/, /2/ and the comparison method for Q-measurement /3/ to establish a set of quickly performed swept frequency techniques for the accurate evaluation of all cavity parameters.

The author gratefully acknowledges financial support from the Fonds zur Förderung der wissenschaftlichen Forschung, Vienna, Austria. This work was carried out as part of the project "Plasma- und Halbleiterforschung in Elektrotechnik und Physik".

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